No. 2913

PURCHASING POWER PARITY
AND THE REAL EXCHANGE RATE

Lucio Sarno and Mark P Taylor

INTERNATIONAL MACROECONOMICS
PURCHASING POWER PARITY
AND THE REAL EXCHANGE RATE

Lucio Sarno, Warwick Business School and CEPR
Mark P Taylor, Warwick Business School and CEPR

Discussion Paper No. 2913
August 2001

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

This Discussion Paper is issued under the auspices of the Centre’s research programme in International Macroeconomics. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as a private educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions. Institutional (core) finance for the Centre has been provided through major grants from the Economic and Social Research Council, under which an ESRC Resource Centre operates within CEPR; the Esmée Fairbairn Charitable Trust; and the Bank of England. These organizations do not give prior review to the Centre’s publications, nor do they necessarily endorse the views expressed therein.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Lucio Sarno and Mark P Taylor
ABSTRACT

Purchasing Power Parity and the Real Exchange Rate*

We assess the progress made by the profession in understanding real exchange rate behaviour, through a selective and critical but nonetheless expository review of the literature. Our reading of the literature leads us to the main conclusions that purchasing power parity might be viewed as a valid long-run international parity condition when applied to bilateral exchange rates among major industrialized countries and that mean reversion in real exchange rates displays significant non-linearities. Further work investigating the effects of real shocks on the long-run equilibrium level also seems warranted.

JEL Classification: F31
Keywords: foreign exchange, purchasing power parity and real exchange rate

Lucio Sarno
Department of Finance
Warwick Business School
University of Warwick
Coventry
CV4 7AL
Tel: (44 24) 7652 8219
Fax: (44 24) 7657 2871
Email: lucio.sarno@warwick.ac.uk

Mark P Taylor
Department of Finance
Warwick Business School
University of Warwick
Coventry
CV4 7AL
Tel: (44 24) 7657 3008
Fax: (44 24) 7652 3032
Email: mark.taylor@wbs.warwick.ac.uk

* This Paper is forthcoming in the International Monetary Fund Staff Papers and was partly written while Lucio Sarno was a Visiting Scholar at the Federal Reserve Bank of St Louis. The authors are grateful to Bob Flood (editor), two anonymous referees and Jim Lothian for constructive criticisms and comments on a previous version of this Paper. Any remaining errors are solely the responsibility of the authors.

Submitted 13 July 2001
NON-TECHNICAL SUMMARY

The purchasing power parity (PPP) exchange rate is the exchange rate between two currencies which would equate the two relevant national price levels if expressed in a common currency at that rate, so that the purchasing power of a unit of one currency would be the same in both economies. This concept of PPP is often termed absolute PPP. Relative PPP is said to hold when the rate of depreciation of one currency relative to another matches the difference in aggregate price inflation between the two countries concerned. If the nominal exchange rate is defined simply as the price of one currency in terms of another, then the real exchange rate is the nominal exchange rate adjusted for relative national price level differences. When PPP holds, the real exchange rate is a constant, so that movements in the real exchange rate represent deviations from PPP. Hence, a discussion of the real exchange rate is tantamount to a discussion of PPP.

Although the term ‘purchasing power parity’ was coined as recently as eighty years ago (Cassel, 1918), it has a very much longer history in economics. While very few contemporary economists would hold that PPP holds continuously in the real world, ‘most instinctively believe in some variant of purchasing power parity as an anchor for long-run real exchange rates’ (Rogoff, 1996), and indeed the implication or assumption of much reasoning in international macroeconomics is that some form of PPP holds at least as a long-run relationship. Moreover, estimates of PPP exchange rates are important for practical purposes such as determining the degree of misalignment of the nominal exchange rate and the appropriate policy response, the setting of exchange rate parities, and the international comparison of national income levels. It is not surprising, therefore, that a large literature on PPP, both academic and policy related, has evolved.

This Paper summarizes the present authors’ reading of recent research on PPP and real exchange rates – what we have learned and what the agenda is for future research. Within the vast literature on PPP and real exchange rates, professional opinion concerning the validity of PPP between the currencies of the major industrialized countries, in both the short and long run, appears to have shifted several times in the post-war period. If there is an emerging consensus at the present time, it is probably reverting towards the view that long-run PPP does have some validity, at least for the major exchange rates, although a number of puzzles have yet to be resolved conclusively.

In our view, a promising strand of research, which goes some way towards resolving both fundamental puzzles in this literature – namely whether PPP holds and whether one can reconcile the persistence of real exchange rates with their observed high volatility – has investigated the role of non-linearities in real exchange rate adjustment toward long-run equilibrium. For example,
Taylor, Peel and Sarno (2001) provide evidence of non-linear mean reversion in a number of major real exchange rates during the post-Bretton Woods period such that real exchange rates behave more like unit root processes the closer they are to long-run equilibrium and, conversely, become more mean reverting the further they are from equilibrium. Moreover, while small shocks to the real exchange rate around equilibrium will be highly persistent, larger shocks mean revert much faster than the ‘glacial rates’ previously reported for linear models (Rogoff, 1996).
The purchasing power parity (PPP) exchange rate is the exchange rate between two currencies which would equate the two relevant national price levels if expressed in a common currency at that rate, so that the purchasing power of a unit of one currency would be the same in both economies. This concept of PPP is often termed absolute PPP. Relative PPP is said to hold when the rate of depreciation of one currency relative to another matches the difference in aggregate price inflation between the two countries concerned. If the nominal exchange rate is defined simply as the price of one currency in terms of another, then the real exchange rate is the nominal exchange rate adjusted for relative national price level differences. When PPP holds, the real exchange rate is a constant so that movements in the real exchange rate represent deviations from PPP. Hence, a discussion of the real exchange rate is tantamount to a discussion of PPP.

Although the term ‘purchasing power parity’ was coined as recently as eighty years ago (Cassel, 1918), it has a very much longer history in economics.\(^1\) While very few contemporary economists would hold that PPP holds continuously in the real world, ‘most instinctively believe in some variant of purchasing power parity as an anchor for long-run real exchange rates’ (Rogoff, 1996), and indeed the implication or assumption of much reasoning in international macroeconomics is that some form of PPP holds at least as a long-run relationship.\(^2\) Moreover, estimates of PPP exchange rates are important for practical purposes such as determining the degree of misalignment of the nominal exchange rate and the appropriate policy response, the setting of exchange rate parities, and the international comparison of national income levels. It is not surprising, therefore, that a large literature on PPP, both academic and policy related, has evolved.

This paper summarizes the present authors’ reading of recent research on PPP and real exchange rates - what we have learned and what the agenda is for future research.\(^3\)

1 PPP, the law of one price and price indices

The law of one price (LOP) is the fundamental building block of the PPP condition. Formally, the LOP in its absolute version may be written as:

\[
P_{i,t} = S_t P_{i,t}^* \quad i = 1, 2, \ldots, N
\]

where \(P_{i,t}\) denotes the price of good \(i\) in terms of the domestic currency at time \(t\), \(P_{i,t}^*\) is the price of good \(i\) in terms of the foreign currency at time \(t\), and \(S_t\) is the nominal exchange rate expressed as the domestic price of the foreign currency at time \(t\). According to equation (1), the absolute version of the LOP essentially postulates that the same good should have the same price across countries if prices are expressed in terms of the same currency of denomination. The basic argument why the LOP should hold is generally based on the idea of frictionless goods arbitrage.

---

\(^1\) The origins of the concept of purchasing power parity have been traced to the writings of scholars from the University of Salamanca in the fifteenth and sixteenth centuries (see e.g. Officer, 1982). Interestingly, the rise in interest in the concept at this time appears to be linked to the prohibition of usury by the Catholic Church. By lending in foreign currency, lenders could justify interest payments by reference to movements in PPP. Thus, Domingo de Banىez could write in 1584: “... one party may lawfully agree to repay a large sum to another, corresponding to the amount required to buy the same parcel of goods that the latter might have bought if he had not delivered his money in exchange”. For a description of the economic thought of the Salamanca School see, for example, Grice-Hutchison (1952, 1975) and Lothian (1975a).\(^2\) This is true of both of the traditional international macroeconomic analysis (e.g. Dornbusch, 1980) and ‘new’ open economy models based on an intertemporal optimizing framework (Obstfeld and Rogoff, 1995, 1996; Lane, 2001; Sarno, 2001).\(^3\) For previous surveys of the PPP literature, see, among others, Breuer (1994), Bleay and Mizen (1995), Froot and Rogoff (1995), Rogoff (1996) and, in the context of a more general survey, Taylor (1995).
In its relative version, the LOP postulates the relatively weaker condition:

$$\frac{P_{i,t+1}^* S_{t+1}}{P_{i,t+1}} = \frac{P_i^* S_t}{P_{i,t}}, \quad i = 1, 2, \ldots, N.$$  \hspace{1cm} (2)

Obviously, the absolute LOP implies the relative LOP, but not vice versa.

Clearly, the LOP can be adequately tested only if goods produced internationally are perfect substitutes. If this is the case, in fact, then the condition of no profitable arbitrage should ensure equality of prices in highly integrated goods markets. Nevertheless, the presence of any sort of tariffs, transport costs and other non-tariff barriers and duties would induce a violation of the no-arbitrage condition and, inevitably, of the LOP. Also, the assumption of perfect substitutability between goods across different countries is crucial for verifying the LOP. In general, however, product differentiation across countries creates a wedge between domestic and foreign prices of a product which is proportional to the freedom of tradability of the good itself.\footnote{An example often used in the literature is the product differentiation of MacDonald’s hamburgers across countries. An example of a good for which the LOP may be expected to hold is gold and other internationally traded commodities (see Rogoff, 1996).}

Formally, by summing up all the traded goods in each country, the absolute version of the PPP hypothesis requires:

$$\sum_{i=1}^{N} \alpha_i P_{i,t} = S_t \sum_{i=1}^{N} \alpha_i P_i^*,$$  \hspace{1cm} (3)

where the weights in the summation satisfy $\sum_{i=1}^{N} \alpha_i = 1$. Alternatively, if the price indices are constructed using a geometric index, then we must form the weighted sum after taking logarithms:

$$\sum_{i=1}^{N} \gamma_i p_{i,t} = s_t + \sum_{i=1}^{N} \gamma_i P_i^*,$$  \hspace{1cm} (4)

where the geometric weights in the summation satisfy $\sum_{i=1}^{N} \gamma_i = 1$ and lower case letters denote logarithms. The weights $\alpha_i$ or $\gamma_i$ are based on a national price index and, according to the seminal, Cassellian formulation of PPP, the consumer price index (CPI). If the national price levels are $P_t$ and $P_t^*$ or, in logarithms, $p_t$ and $p_t^*$, then (according to whether the arithmetic or geometric index is used), we can use (3) or (4) to derive the (absolute) PPP condition:

$$s_t = p_t - p_t^*.$$  \hspace{1cm} (5)

From equation (5) it is easily seen that the real exchange rate, defined here in logarithmic form:

$$q_t = s_t - p_t + p_t^*,$$  \hspace{1cm} (6)

may be viewed as a measure of the deviation from PPP.

Clearly, deriving PPP from the LOP introduces a range of index number problems. For example, equations (3) and (4) implicitly assume that the same weights are relevant in each country, whereas price index weights will typically differ across different countries (perhaps even being zero in one country and non-zero in another for some goods and services) and will also tend to shift through time. In practice, researchers often assume that PPP should hold approximately using the price indices of each country. In the geometric index case, for example, we can rearrange (4) to yield:

$$\sum_{i=1}^{N} \gamma_i p_{i,t} = s_t + \sum_{i=1}^{N} \gamma_i P_i^* + \sum_{i=1}^{N} (\gamma_i - \gamma_i^*) p_{i,t}^*$$  \hspace{1cm} (7)
or

\[
\sum_{i=1}^{N} \gamma_i p_{t,i} = s_t + \sum_{i=1}^{N} \gamma_i p_{t,i}^2 + u_t, \tag{8}
\]

where the \( \gamma_i \) denote the weights in the foreign price index. Clearly, the greater the disparity between the relevant national price indices, the greater the apparent disparity - represented by \( u_t \) - from aggregate PPP even when the LOP holds for individual goods. Note, however, that because the geometric price indices are homogeneous of degree one (i.e. an equiproportionate increase in all prices will raise the overall price level by the same proportion) then differences in weights across countries will matter less where price impulses affect all goods and services more or less homogeneously. An \( x \) percent increase in all prices in the foreign country will lead, for example, to an \( x \) percent increase in the foreign price level and the right hand side of (8) will be augmented by \( x \) and the change in the \( u_t \) term will be zero. Thus, assuming domestic prices are constant, an \( x \) percent appreciation of the domestic currency is required in order to restore equilibrium.

A similar analysis may be applied when some goods and services are non-traded. Suppose that the LOP applies only among traded goods. An \( x \) percent increase in all foreign traded goods prices implies, other things equal, an \( x \) percent appreciation of the domestic currency. But if there is also an \( x \) percent rise in all \( non-traded \) foreign goods prices, the PPP condition based on individual national price indices will also imply an \( x \) percent exchange rate movement.

In practice, it is more common for national statistical bureaus to use arithmetic rather than geometric price indices, although deviations from measured PPP arising from this source are not likely to be large. Considerable differences may arise, however, where price impulses impinge heterogeneously across the various goods and services in an economy and, in particular, where price inflation differs between the traded and non-traded goods sectors. A particular example of this - the Harrod-Balassa-Samuelson effect - is discussed below.

The choice of the appropriate price index to be used in implementing absolute PPP has been the object of a long debate in the literature, going back at least as far as Keynes (1923). All commonly used price measures include some proportion of nontraded goods, which may induce rejection of PPP or at least of the conditions of homogeneity and proportionality (discussed below) required by PPP. Thus, many attempts exist in the literature to construct appropriate price measures for testing PPP. The most influential work in this context has been carried out by Summers and Heston (1991), who developed the “International Comparison Programme” (ICP) data set, which reports estimates of absolute PPP for a long sample period and a number of countries, using a common basket of goods across countries. The ICP is not, however, of great practical help in much empirical work since it is constructed at infrequent and large time intervals and, for certain time periods, data are only available for several countries. Moreover, since extensive use of extrapolation has been made in order to solve this problem, the data presented in the ICP becomes partially artificial, somehow losing reliability. Overall, therefore, price indices made available by official sources still remain the basis commonly used for implementing absolute PPP, despite the discussed limitations.

In general, however, the difficulty in finding evidence strongly supportive of PPP and the difficulties encountered in moving from the LOP to PPP has provided a strong motivation for researchers to investigate the LOP empirically.
2 Empirical evidence on the LOP

Recent econometric tests of the LOP have often been motivated as a reaction to the rejection of PPP during the recent floating exchange rate regime, which we discuss further below. In general, econometric studies suggest rejection of the LOP for a very broad range of goods and provide strong empirical evidence both that deviations from the LOP are highly volatile and that the volatility of relative prices is considerably lower than the volatility of nominal exchange rates. Some recent studies, however, provide evidence that departures from the LOP may dissipate over time when they are modelled using a nonlinear framework.

2.1 The empirical literature on the LOP using national-level data

At least two influential empirical studies on the LOP were executed in the 1970s. First, Isard (1977) uses disaggregated data for a number of traded goods (chemical products, paper and glass products among others) and for a number of countries, providing strong empirical evidence that the deviations from the LOP are large and persistent and appear to be highly correlated with exchange rate movements. Second, Richardson (1978) finds very similar results to Isard, by using data for 4- and 7-digit standard industrial classification (SIC) categories.

Giovannini (1988) uses a partial equilibrium model of the determination of domestic and export prices by a monopolistic competitive firm and argues that the stochastic properties of deviations from the LOP are strongly affected by the currency of denomination of export prices. In particular, Giovannini uses data on domestic and dollar export prices of Japanese goods and provides evidence that deviations from the LOP - found to be large not only for sophisticated manufacturing goods but also for commodities such as screws, nuts and bolts - are mainly due to exchange rate movements, consistent with the earlier relevant literature (see also Benninga and Protopapadakis, 1988; Goodwin, Gremes and Wohlgemant, 1990; Bui and Pippinger, 1990; Fraser, Taylor and Webster, 1991; Goodwin, 1992).

Some of the most influential and convincing work in testing for the LOP is provided by Knetter (1989, 1993). Knetter uses high-quality disaggregated data (7-digit) and provides evidence that large and persistent price differentials exist for traded goods exported to multiple destinations (e.g. for German beer exported to the UK as compared to the US). Another interesting study in this context is due to Engel (1993), who uncovers a strong empirical regularity: the consumer price of a good relative to a different good within a country tends to be much less variable than the price of that good relative to a similar good in another country. This fact holds for all goods except very simple, homogeneous products. Engel suggests that models of real exchange rates are likely to have predictions regarding this relation, so this fact may provide a useful gauge for discriminating among models.

2.2 The empirical literature on the LOP using city-level data

Parsley and Wei (1996) look for convergence towards the LOP in the absence of trade barriers or nominal exchange rate fluctuations by analyzing a panel of 51 prices from 48 cities in the United States. They find convergence rates substantially higher than typically found in cross-country data, that convergence occurs faster for larger price differences and that rates of convergence are slower for cities further apart. Extending this line of research, Engel and Rogers (1996) use CPI

\[5\text{See also the related work of Herquem (1994), Chen and Kneze (1995), and Dumas, Jennrgren and Nashu (1995).}\]
data for both US and Canadian cities and for fourteen categories of consumer prices in order to analyze the stochastic properties of deviations from the LOP. The authors provide evidence that the distance between cities can explain a considerable amount of the price differential of similar goods in different cities of the same country. Nevertheless, the price differentials are considerably larger for two cities across different countries relative to two equidistant cities in the same country. The estimates of Engel and Rogers suggest that crossing the national border - the so-called “border effect” - increases the volatility of price differentials by the same order of magnitude which would be generated by the addition of 2,500 to 23,000 extra miles between the cities considered. Rogers and Jenkins (1995) find similar results to Engel (1993), providing evidence that the “border effect” is effective in increasing not only the volatility of price differentials but also their persistence.

2.3 Pricing to market

One story for rationalizing the rejection of the LOP comes from the “pricing to market” (PTM) theory of Krugman (1987) and Dornbusch (1987). Following the developments of theories of imperfect competition and trade, the main feature of this theory is that the same good can be given a different price in different countries when oligopolistic firms are supplying it. This is feasible because there are many industries which can supply separate licences for the sale of their goods at home and abroad.6 At the empirical level, Knetter (1989, 1993) finds that PTM is very important for German and Japanese firms relative to US companies and that it is a strategy used for a very broad range of goods.7

Kasa (1992) argues, however, that the rationale underlying PTM is not price discrimination, as proposed by Krugman and Dornbusch. Kasa argues that PTM is better rationalized by an adjustment cost framework, that is a model in which firms face some sort of menu costs or a model in which consumers face fixed costs when switching between different products (see also Froot and Klemperer, 1989).

In an interesting study, Ghosh and Wolf (1994) examine the statistical properties and the determinants of changes in the cover price of The Economist newspaper across twelve countries during the recent float. They show that standard tests of PTM may fail to discriminate the alternative hypothesis of menu costs. Their findings suggest a strong violation of the LOP and are consistent with menu-cost-driven pricing behavior. More recently, Haskel and Wolf (2001), use retail transaction prices for a multinational retailer to examine the extent and permanence of violations of LOP. For identical products, Haskel and Wolf find typical deviations of twenty to fifty percent, though there is muted evidence for convergence over time. The authors argue that such differences might be due to differences in local costs. If so, relative prices of similar products (round versus square mirrors) should be equal across countries. In fact, relative prices vary significantly across very similar goods within a product group; also, the ordering of common currency prices often differs for similar products, suggesting that differences in local distribution costs, local taxes, and probably tariffs do not explain the price pattern, leaving strategic pricing or other factors resulting in varying markups as alternative explanations for the observed divergences.8

---

6 Froot and Rogoff (1995) note how the PTM theory not only can explain the long-run deviations from the LOP, but has important implications for the transmission mechanism of disturbances from the money market in the presence of nominal rigidities (see also Marston, 1990).

7 A potential explanation of this finding is provided by Rangan and Lawrence (1993) who argue that, since US firms sell a large part of their exports through subsidiaries, the PTM by US firms may occur at subsidiary level. In this case, the comparison executed by Knetter may lead to an underestimation of the importance of PTM by US firms.

8 Another issue which is worth noting is the possibility that the failure of the LOP may be explained by instit-
2.4 Nonlinearities in deviations from the LOP

Among the possible explanations of the violation of the LOP suggested by the literature, transport costs, tariffs and nontariff barriers play a dominant role. An estimate of the wedge driven by the costs of transportation is given, for example, by the International Monetary Fund (IMF, 1994): the difference between the value of world exports computed as “free on board” (FOB) and the value of world imports charged in full, or cost, insurance and freight (CIF) is estimated at about ten percent and is found to be highly variable across countries. Moreover, the presence of significant nontraded components in the price indices used by the empirical literature may induce violations of the LOP. Even if the wholesale price index (WPI) includes a smaller nontraded component relative to the consumer price index, it still includes a significant nontraded component (e.g. the cost of labor employed and insurance). Moreover, even if tariffs have been considerably reduced over time across major industrialized countries, nontariff barriers are still very significant. Governments of many countries often intervene in trade across borders using nontariff barriers in a way that they do not use within their borders (for example in the form of strict inspection requirements; see Knetter, 1994; Feenstra, 1995; Rogoff, 1996; Feenstra and Kendall, 1997).

Frictions in international arbitrage have important implications and, in particular, imply potential nonlinearities in the deviations from the LOP. The idea that there may be nonlinearities in goods arbitrage dates at least from Heckscher (1916), who suggested that there may be significant deviations from the LOP due to international transaction costs between spatially separated markets. A similar viewpoint can be discerned in the writings of Cassel (e.g. Cassel, 1922) and, to a greater or lesser extent, in other earlier writers (Officer, 1982). More recently, a number of authors have developed theoretical models of nonlinear real exchange rate adjustment arising from transaction costs in international arbitrage (e.g. Benninga and Protopapadakis, 1988; Williams and Wright, 1991; Dumas, 1992; Sercu, Uppal and Van Hulle, 1995; O’Connell, 1997; Ohanian and Stockman, 1997). In most of these models, proportional or ‘iceberg’ transport costs (‘iceberg’ because a fraction of goods are presumed to ‘melt’ when shipped) create a band for the real exchange rate within which the marginal cost of arbitrage exceeds the marginal benefit. Assuming instantaneous goods arbitrage at the edges of the band then typically implies that the thresholds become reflecting barriers.

Drawing on recent work on the theory of investment under uncertainty, some of these studies show that the thresholds should be interpreted more broadly than as simply reflecting shipping costs and trade barriers per se, but also as resulting from the sunk costs of international arbitrage and the resulting tendency for traders to wait for sufficiently large arbitrage opportunities to open up before entering the market (see in particular Dumas, 1992; also Dixit, 1989, and Krugman, 1989). O’Connell and Wei (1997) extend the iceberg model to allow for fixed as well as proportional costs of arbitrage. This results in a two-threshold model where the real exchange rate is reset by arbitrage to an upper or lower inner threshold whenever it hits the corresponding outer threshold. Intuitively, arbitrage will be heavy once it is profitable enough to outweigh the initial fixed cost, but will stop short of returning the real rate to the PPP level because of the proportional arbitrage costs. Coleman (1995) suggests that the assumption of instantaneous trade should be replaced with the presumption that it takes time to ship goods. In this model, transport costs again create a band of no arbitrage for the real exchange rate, but the exchange rate can stay beyond the thresholds. Once beyond the upper or lower threshold, the real rate becomes increasingly mean

---

*(normal factors typical of this century which have increased the persistence of deviations from the LOP. Nevertheless, Froot, Kim and Rogoff (1995), using data on prices for grains and other dairy goods in England and Holland for a span of data which goes from the fourteenth to the twentieth century, find that the volatility of the LOP is quite stable during the whole period, regardless of the many regime shifts during the sample.)*
reverting with the distance from the threshold. Within the transaction costs band, when no trade takes place, the process is divergent so that the exchange rate spends most of the time away from parity.

Some empirical evidence of the effect of transaction costs in this context is provided by Davutyan and Pippenger (1990). More recently, Obstfeld and Taylor (1997) have investigated the nonlinear nature of the adjustment process in terms of a threshold autoregressive (TAR) model (Tong, 1990). The TAR model allows for a transaction costs band within which no adjustment in deviations from the LOP takes place - so that deviations may exhibit unit root behavior - while outside of the band, as goods arbitrage becomes profitable and its effects are felt, the process switches abruptly to become stationary autoregressive. Obstfeld and Taylor provide evidence that TAR models work well when applied to disaggregated data, and yield estimates in which the thresholds correspond to popular rough estimates of the order of magnitude of actual transport costs.

More recently, Sarno, Taylor and Chowdhury (2001) test empirically the validity of the law of one price using data for five major bilateral US dollar exchange rates and nine goods sectors during the recent floating exchange rate regime since the early 1970s. Using threshold autoregressive models, the authors find strong evidence of nonlinear mean reversion in deviations from the law of one price with plausible convergence speeds. Consistent with theoretical arguments on international goods markets arbitrage under transactions costs and with the emerging strand of empirical literature cited above, their results contribute towards forming a consensus view in favor of discrete regime switching in deviations from the LOP and the presence of differing nonzero transactions costs across a broad range of goods and countries. In particular, it appears that goods markets between the US and Japan have lower transactions costs than between the US and Europe, consistent with the findings of Obstfeld and Taylor (1997). In general, adjustment towards the LOP is observed to be fairly fast although the estimated delay parameter, which measures the timing of the reaction of market participants to deviations from the LOP, is estimated to be longer than one might perhaps expect. Also, these results suggest that deviations from the LOP may be somewhat sticky (given the delay parameter is on average larger than four quarters), but they are not as persistent as a large literature has hitherto suggested.

3 Empirical evidence on PPP

The empirical evidence on PPP is extremely large and the sophistication of the testing procedures employed has developed pari passu with advances in econometric techniques. Hence, it is useful to separate the enormous empirical evidence on PPP into six different stages: the early empirical literature on PPP; tests of the random walk hypothesis for the real exchange rate; cointegration studies; long-span studies; panel data studies; and, finally, studies employing nonlinear econometric techniques.

3.1 The early empirical literature on PPP

Absolute PPP implies that the nominal exchange rate is equal to the ratio of the two relevant national price levels. Relative PPP posits that changes in the exchange rate are equal to changes in relative national prices. The early empirical literature - until the late 1970s - on testing PPP is based on estimates of equations of the form:

\[ s_t = \alpha + \beta p_t + \beta^* p_t^* + \omega_t, \]  

(9)
where \( \omega \) is a disturbance term. A test of the restrictions \( \beta = 1, \beta^* = -1 \) would be interpreted as a test of absolute PPP, whilst a test of the same restrictions applied to the equation with the variables in first differences would be interpreted as a test of relative PPP. In particular, a distinction is often made between the test that \( \beta \) and \( \beta^* \) are equal and of opposite sign - the symmetry condition - and the test that they are equal to unity and minus unity respectively - the proportionality condition.

In the earlier relevant literature, researchers did not introduce dynamics in the estimated equation in such a way as to distinguish between short-run and long-run effects, even if it was recognized by researchers that PPP is only expected to hold in the long run. Nevertheless, the empirical literature based on estimation of equations of the form (9) generally suggest rejection of the PPP hypothesis. In an influential study, however, Frenkel (1978), obtains estimates of \( \beta \) and \( \beta^* \) very close to plus and minus unity on data for high inflation countries, suggesting that PPP represents an important benchmark in long-run exchange rate modelling. Several drawbacks affect, however, this approach. First, Frenkel does not investigate the stochastic properties of the residuals and in particular does not test for stationarity. If the residuals are not stationary, in fact, part of the shocks impinging upon the real exchange rate will be permanent, i.e. PPP is violated. Second, apart from hyperinflationary economies, PPP tends to be strongly rejected on the basis of estimates of equations such as (9). Frenkel argues, however, that the rejection of PPP may be due only to temporary real shocks and price stickiness in the goods market, but convergence to PPP is expected to occur in the long run.\(^9\)

The crucial problem is, however, that this early literature does not investigate the stationarity of the residuals in the estimated equation. If both nominal exchange rates and relative prices are nonstationary variables (and are not cointegrated), then (9) is a spurious regression and conventional OLS-based statistical inference is invalid (Granger and Newbold, 1974). If the error term in (9) is stationary, however, then a strong long-run linear relationship exists between exchange rates and relative prices, but conventional statistical inference is still invalid because of the bias present in the estimated standard errors (Engle and Granger, 1987; Banerjee, Dolado, Hendry and Smith, 1986).

The next stage in the development of this literature was explicitly to address the issue of nonstationarity of the variables under consideration, starting with an analysis of whether the real exchange rate itself is stationary - implying evidence of long-run PPP - or whether it tends to follow a unit root process - implying absence of any tendency to converge on a long-run equilibrium level.

### 3.2 Tests for a unit root in the real exchange rate

Recall that the real exchange rate in its logarithmic form may be written as:

\[
q_t = s_t + p_t^e + p_t
\]

The approach taken by the second stage of tests of PPP undertaken by the empirical literature is based on testing for the nonstationarity of the real exchange rate. Early studies taking this approach include - among others - Roll (1979), Adler and Lehmann (1983), Hakkio (1984), Edison (1985), Frankel (1986), Huizinga (1987), and Meese and Rogoff (1988). From the mid to late 1980s

\(^9\)Another problem in testing PPP on the basis of estimates of equation (9) is the endogeneity of both nominal exchange rates and price levels: instead the choice of the variable to be put on the left-hand side of (9) is arbitrary. Kregman (1978) constructs a flexible-price exchange rate model in which the domestic monetary authorities intervene against real shocks using expansionary monetary policies, therefore inducing inflation. The model is estimated by instrumental variables (IV) and ordinary least squares (OLS). The IV estimates of \( \beta \) and \( \beta^* \) are closer to unity in absolute value relative to the OLS estimates, but PPP is still rejected (see also Frenkel, 1981).
onward, a basic standard approach has been to employ a variant of the augmented Dickey-Fuller (ADF) test for a unit root in the process driving the real rate. This is generally based on an auxiliary regression of the general form:

\[ \Delta q_t = \gamma_0 + \gamma_1 t + \gamma_2 q_{t-1} + \Xi(L) \Delta q_{t-1} + \epsilon_t, \]  

(11)

where \( \Xi(L) \) denotes a \( p \)-th order polynomial in the lag operator \( L \), and \( \epsilon_t \) is a white noise process. Testing the null hypothesis that \( \gamma_2 = 0 \), via an ADF test, is tantamount to testing for a single unit root in the data generating process for \( q_t \) and would imply no long-run equilibrium level for \( q_t \). The alternative hypothesis that PPP holds requires that \( \gamma_1 < 0 \). A variant of this approach is to use a modified version of this test to allow for non-Gaussian disturbances (Phillips, 1986; Phillips and Perron, 1988).

A second approach for testing for nonstationarity of the real exchange rate involves variance ratio tests. In this case the persistence of the real exchange rate is measured using a simple nonparametric test, due originally to Cochrane (1988), \( z(k) \):

\[ z(k) = {1 \over k} \text{Var}(q_t - q_{t-h}) {k \over \text{Var}(q_t - q_{t-1})}, \]  

(12)

where \( k \) is a positive integer and \( \text{Var} \) stands for variance. If the real exchange rate follows a random walk, then the ratio in (12) should equal unity, since the variance of a \( k \)-period change should be \( k \) times the variance of a one-period change. By contrast, if the real exchange rate exhibits mean reversion, the ratio \( z(k) \) should be in the range between zero and unity.

A third approach involves employing the techniques developed by the literature on fractional integration, since these techniques allow the researcher to consider a broader range of stationary processes under the alternative hypothesis relative to conventional unit root tests. Formally, the real exchange rate process may be represented as:

\[ \Phi(L)(1 - L)^d q_t = \zeta(L) w_t, \]  

(13)

where \( \Phi(L) \) and \( \zeta(L) \) are both polynomials in \( L \) with roots lying outside the unit circle, and \( w_t \) is a white noise process. Under this approach the parameter \( d \) is allowed to lie in the continuous interval between zero and unity. Fractionally integrated processes are more persistent than pure autoregressive-moving-average (ARMA) processes, but are still stationary. If \( d = 0 \), then the real exchange rate simply follows an ARMA process. On the other hand, if \( d, \Phi(L) \) and \( \zeta(L) \) all equal unity, the real exchange rate follows a random walk (see Diebold, Husted and Rush, 1991; Cheung and Lai, 1993a).

Empirical studies employing tests of the type described in this section for testing PPP during the recent float generally cannot reject the random walk hypothesis for the real exchange rates of the currencies of all the major industrialized countries against one another, therefore suggesting that deviations from PPP are permanent (see also Enders, 1988; Taylor, 1988; Mark, 1990; Edison and Pauls, 1993). Two exceptions are Huizinga (1987) who uses variance ratio tests and data for dollar exchange rates against a number of currencies for sample periods shorter than two years; and Chowdhury and Sdogati (1993), who analyze the European Monetary System (EMS) period 1979-1990 and find support for PPP for real exchange rates when expressed \textit{vis-a-vis} the German mark, but not when expressed \textit{vis-a-vis} the US dollar.\footnote{Another result supportive of PPP is due to Whitt (1992). Whitt uses a Bayesian unit root test due to Sims (1988), and is able to reject the null hypothesis that the real exchange rate follows a random walk for a number of countries and for both the pre- and the post-Bretton Woods period.}
3.3 Cointegration studies of PPP

Cointegration, as originally developed by Engle and Granger (1987), seems to be an ideal approach to testing for PPP. While short-run deviations from the equilibrium level implied by long-run PPP are admissible, a necessary condition for long-run PPP to hold is that the “equilibrium error” (Granger 1986) $q_t$ is stationary over time. If this is not the case, then the nominal exchange rate and the relative price will permanently tend to deviate from each other. Cointegration analysis tells us that any two nonstationary series, which are found to be integrated of the same order are cointegrated if a linear combination of the two exists which is itself stationary. If this is the case, then the nonstationarity of one series exactly offsets the nonstationarity of the other and a long-run relationship is established between the two variables. In our context, if both the nominal exchange rate $s_t$ and the relative price $\pi_t$ have a stationary, invertible, non-deterministic ARMA representation after differencing $d$ times, i.e. they are both integrated of order $d$ or $I(d)$, then the linear combination:

$$s_t + \kappa \pi_t = z_t$$  \hspace{1cm} (14)

will in general found to be $I(d)$ as well, if the real exchange rate has a random walk component. Nevertheless, if a cointegrating parameter $\alpha$ exists such that $q_t$ is integrated of order $I(d-c)$, $c > 0$, then the nominal exchange rate and the relative price are cointegrated of order $d-c$, or $CI(d,c)$.

In the context of PPP testing we want $d = c = 1$, that is $s_t$ and $\pi_t$ are both $I(1)$ variables, but $z_t$ is mean-reverting. In this case one may feel confident that a strong long-run relationship exists between the two variables considered, since they share a common stochastic trend (Stock and Watson, 1988) and “cointegration of a pair of variables is at least a necessary condition for them to have a stable long-run (linear) relationship” (Taylor 1988; Taylor and McMahon, 1988).

However, if the no-cointegration hypothesis cannot be rejected, then the estimated regression is just a “spurious” one and has no economic meaning: the analysis is subject to the same drawbacks discussed above. Given that no bounded combination of the levels exists, then the error term in the regression must be nonstationary under the null hypothesis.

The main difference in using cointegration in testing for PPP rather than testing for the nonstationarity of the real exchange rate is that the symmetry and proportionality conditions are not imposed and cannot be tested easily given the bias in the estimated standard errors. Rationales for the rejection of the symmetry and proportionality conditions, based on considerations of measurement errors (in particular systematic differences between actual measured price indices and those theoretically relevant for PPP calculations) and barriers to trade, are provided by, inter alios, Taylor (1988), Fisher and Park (1991) and Cheung and Lai (1993a, 1993b).

The Johansen (1988, 1991) maximum likelihood estimator circumvents these problems and enables us to test for the presence of multiple cointegrating vectors. Johansen shows how to test for linear restrictions on the parameters of the cointegrating vectors and this is of great interest because it makes it possible to test the symmetry and proportionality conditions exactly.\footnote{It is also possible to circumvent the problem by simply estimating the regression of the nominal exchange rate on the relative price by fully-modified OLS (FM OLS), due to Phillips and Hansen (1990), instead of OLS, since a correction is made for the problem of the bias in the standard errors. Alternatively, one could employ the dynamic OLS (DOLS) estimator developed by Stock and Watson (1993).}

Earlier cointegration studies generally reported the absence of significant mean reversion of the exchange rate towards PPP for the recent floating experience (Taylor, 1988; Mark, 1990), but were supportive of reversion towards PPP for the interwar float (Taylor and McMahon, 1988), for the 1950s US-Canadian float (McNown and Wallace, 1989), and for the exchange rates of high-inflation countries (Choudhry, McNown and Wallace, 1991). More recent applied work on long-run PPP...
among the major industrialized economies has, however, been more favorable towards the long-run PPP hypothesis for the recent float (e.g. Corbae and Ouliaris 1988; Kim, 1990; Cheung and Lai, 1993a, 1993b).

Overall, cointegration studies highlight some important features of the data. The null hypothesis of no-cointegration is more easily rejected when in the sample period considered the exchange rates are fixed rather than floating. Also, interestingly, stronger evidence supporting PPP is suggested when the WPI is used rather than the CPI and, even more so, when the GDP deflator is used. This is easy to explain since the WPI price level contains a relatively smaller nontradables component and represents therefore a better approximation to the ideal price index required by the PPP hypothesis than the CPI and the GDP deflator.\footnote{The argument that PPP should hold better with the WPI than with the CPI goes back to Keynes [1932] and McKinnon [1971]. Nevertheless, note that the CPI-WPI distinction is a subtle one since the two price indices are very highly correlated and differences in their movements are, in general, very difficult to explain.}

Another feature of the data suggested by the cointegration literature is that in bivariate systems cointegration is established more frequently than in trivariate systems and in Engle-Granger two-step procedures. The disappointing finding is, however, that the symmetry and proportionality conditions are very often rejected and the parameters estimated in PPP regressions are often far from the theoretical values. While this result may simply be caused by small-sample bias in the case of two-step cointegration procedures, it is difficult to explain rejections occurring in large samples and in estimates obtained using the Johansen procedure. Thus, the problem may simply be that longer data sets are needed to detect PPP and mean reversion in the real exchange rate. In general, rejection of PPP may be due to lack of power of conventional econometric tests. Some notable attempts to overcome this problem are discussed in the following sections.

### 3.4 The power problem

Following an early warning from Frankel (1986, 1990), a number of authors have noted that the tests typically employed during the 1980s to examine the long-run stability of the real exchange may have very low power to reject a null hypothesis of real exchange rate instability when applied to data for the recent floating rate period alone (e.g. Lothian, 1986, 1998a; Froot and Rogoff, 1995; Lothian and Taylor, 1996, 1997). The argument is that if the real exchange rate is in fact stable in the sense that it tends to revert towards its mean over long periods of time, then examination of just one real exchange rate over a period of twenty-five years or so may not yield enough information to be able to detect slow mean reversion towards PPP.

A straightforward way of illustrating this point is through a simple Monte Carlo experiment. As discussed in the next section, Lothian and Taylor (1996) estimate an AR(1) process for the sterling-dollar and sterling-franc real exchange rates using two centuries of data. For sterling-dollar, they report the following AR(1) model:

\[
q_t = 0.179 + 0.887 q_{t-1} + \hat{\varepsilon}_t, \tag{15}
\]

where \( \hat{\varepsilon}_t \) is the fitted residual, which has an estimated standard error of 7.1 percent and figures in parentheses are estimated standard errors. As discussed below, the estimated first-order autocorrelation coefficient implies a speed of mean reversion - about 11 percent per annum - which is in fact quite typical in the literature employing panel data or long spans of data. Indeed, the 95 percent confidence interval which ranges from about 0.825 to about 0.95 would certainly encompass the range of reported point estimates (see Rogoff, 1996). Hence, we can use this estimated model as
a basis for our Monte Carlo experiments. Accordingly, we simulated data from an artificial data generating process calibrated on this model, for various sample sizes and with the autoregressive coefficient taking the value 0.825, 0.887, and 0.95. In each case, we generated 10,000 artificial data sets of length $T + 100$, where $T$ is the particular sample size, starting with an initial value of $\varrho_0 = 0$. For each artificial data set we then calculated the simple Dickey-Fuller statistic (after discarding the initial 100 data points) and compared this to the 5 percent critical value obtained using the response surface estimates of McKinnon (1991). The proportion of times we were able to reject the null hypothesis of a unit root out of 10,000 cases then gives us the empirical power for that particular sample size and autoregressive coefficient. The resulting empirical power function is tabulated in Table 1.\textsuperscript{13}

<table>
<thead>
<tr>
<th>$T$:</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varrho = 0.950$</td>
<td>4.79</td>
<td>5.70</td>
<td>6.02</td>
<td>7.41</td>
<td>9.81</td>
<td>12.54</td>
<td>21.17</td>
<td>32.81</td>
<td>47.31</td>
</tr>
<tr>
<td>$\varrho = 0.887$</td>
<td>6.33</td>
<td>6.99</td>
<td>7.95</td>
<td>15.13</td>
<td>26.03</td>
<td>41.26</td>
<td>75.13</td>
<td>93.80</td>
<td>99.31</td>
</tr>
<tr>
<td>$\varrho = 0.825$</td>
<td>7.44</td>
<td>9.26</td>
<td>11.41</td>
<td>28.07</td>
<td>53.55</td>
<td>78.08</td>
<td>98.21</td>
<td>99.95</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Notes: $T$ is the sample size; $\varrho$ is the first-order autocorrelation coefficient.

Much of the early work on unit roots and cointegration for real exchange rates was published in the late 1980s and was therefore based on data spanning the fifteen years or so since the period of generalized floating beginning in 1973. As Table 1 shows, however, for the speeds of mean reversion typically recorded in the literature (Froot and Rogoff, 1995; Rogoff, 1996), the probability of rejecting the null hypothesis of a random walk real exchange rate when in fact the real rate is mean reverting would only be somewhere between about 5 and 7.5 percent. Given that we have, of course, only one data set on real exchange rates available, an alternative way of viewing this is to note that if real exchange rates are in fact mean reverting in this fashion, the probability of \textit{never} being able to reject the null hypothesis of a unit root given the available data is in excess of 92 percent when we have only fifteen years of data available. Even with the benefit of the additional ten years or so of data which is now available, however, the power of the test increases only slightly, to a maximum of around 11 percent on the most optimistic view of the speed of mean reversion. Taking the point estimate obtained by Lothian and Taylor (1996) of 0.887, we confirm their finding that “even with a century of data on the sterling-dollar real exchange rate, we would have less than an even chance of rejecting the unit root hypothesis” (Lothian and Taylor, 1996, pp. 950-1). Moreover, even if we consider the extreme lower end of the 95 percent confidence interval of 0.825 for the first-order autocorrelation coefficient, we should still need something like 75 years of data in order to be able to reject the null hypothesis with more than 50 percent probability.\textsuperscript{14}

The Monte Carlo evidence of Shiller and Perron (1985) demonstrates, moreover, that researchers cannot circumvent this problem by increasing the frequency of observation - say from annual to quarterly or monthly - and thereby increasing the number of data points available. Given that, in a spectral analysis sense, we are examining the low frequency components of real exchange rate behavior, this requires a long \textit{span} of data in terms of years in order to improve the power of the

\textsuperscript{13} These results are consistent with those reported in Lothian and Taylor (1997), although the present tabulations are more comprehensive, are based on 10,000 rather than 5,000 simulations, and use the exact 5% critical values calculated using McKinnon’s (1991) response surface estimates.

\textsuperscript{14} Engel (2000), using artificial data calibrated to nominal exchange rates and disaggregated data on prices also shows that standard unit root and cointegration tests applied to real exchange rate data may have significant size biases and also demonstrates that tests of stationarity may have very low power.
This realization led some researchers to do exactly that - i.e. examine the behavior of real exchange rates using very long data sets. An alternative means of increasing test power is to keep the same length of data set (say since 1973) but to test for unit roots jointly using a panel of real exchange rates for a number of countries. This literature is discussed below.\(^\text{16}\)

### 3.5 Long span studies

The first approach considered in the literature to circumvent the low power problem of conventional unit root tests was to employ long span data sets.\(^\text{17}\) For example, using annual data from 1869 to 1984 for the dollar-sterling real exchange rate, Frankel (1986) estimates an AR(1) process for the real rate with an autoregressive parameter of 0.86 and is able to reject the random walk hypothesis. Long-run PPP for the dollar-sterling exchange rate is also examined by Edison (1987) over the period 1890-1978, using an error-correction mechanism (ECM) of the form:

\[
\Delta s_t = \delta_0 + \delta_1 \Delta (p_t - p^*_t) + \delta_2 (s_{t-1} - p_{t-1} + p^*_{t-1}) + u_t, 
\]

which has a long-run constant equilibrium level of real exchange rate. Edison's results provide evidence that PPP holds, but shocks impinging upon the real exchange rate are very persistent and the half-life is about 7.3 years. Glen (1992) also finds mean-reversion of the real exchange rates for nine countries and a half-life of 3.3 years over the sample period 1900-1987 (see also Cheung and Lai, 1994).\(^\text{18}\)

Lothian and Taylor (1996) use two centuries of data on dollar-sterling and franc-sterling real exchange rates and provide indirect evidence supporting PPP in the recent floating period. They cannot find any significant evidence of a structural break between the pre- and post-Bretton Woods period using a Chow test and show that the widespread failure to detect mean reversion in real exchange rates during the recent float may simply be due to the shortness of the sample.

Long span studies have, however, been subject to some criticism in the literature. One criticism relates to the fact that, because of the very long data spans involved, various exchange rate regimes are typically spanned. Also, real shocks may have generated structural breaks or shifts in the equilibrium real exchange rate (see e.g. Hegwood and Papell, 1999). This is, of course, a 'necessary evil' with long-span studies of which researchers are generally aware. Moreover, researchers using long-span data are generally at pains to test for structural breaks (see e.g. Lothian and Taylor, 1996).

Nevertheless, in order to provide a convincing test of real exchange rate stability during the post Bretton Woods period, it is necessary to devise a test using data for that period alone. This provided the impetus for panel data studies of PPP.

\(^{15}\)Similar remarks would apply to variance ratio tests and tests for non-cointegration.

\(^{16}\)Nevertheless, it should be noted that there exists a related problem with testing the null hypothesis of a unit root in the real exchange rate, namely that for any finite sample of data a unit root process may be arbitrarily well approximated by a stationary process. This is an issue relating to finite sample and asymptotic size distortion, rather than power - see, for example, Faust (1996).

\(^{17}\)As discussed above, alternative unit root tests may also be sufficiently powerful to detect mean reversion in real exchange rates. For example, Diebold, Husted and Rush (1991) and Cheung and Lai (1993a) apply fractional integration techniques and find evidence supporting long-run PPP. See also Taylor (2001a).

\(^{18}\)See also Lothian (1990, 1991) for work on the Japanese yen using a long span of data and Lothian (1998b) for a study using nearly four centuries of bimennial data on the Dutch guilder-UK sterling.
3.6 Panel data studies

A different approach undertaken by the literature on testing for PPP in order to circumvent the problem of low-powers displayed by conventional unit root tests is to increase the number of exchange rates under consideration.

The first attempt is due to Hakkio (1984), who employs generalized least squares (GLS) and tests the null hypothesis of nonstationarity using data for a system of four exchange rates. Hakkio cannot rejects, however, the null hypothesis that all real exchange rates under examination follow a random walk.

Abuaf and Jorion (1990) employ a similar approach in that they examine a system of ten AR(1) regressions for real dollar exchange rates where the first-order autocorrelation coefficient is constrained to be equal across rates, taking account of contemporaneous correlations among the disturbances. The estimation is executed employing Zellner’s (1962) “seemingly unrelated” (SUR) estimator, which is basically multivariate GLS using an estimate of the contemporaneous covariance matrix of the disturbances obtained from individual OLS estimation. Thus, Abuaf and Jorion test the null hypothesis that the real exchange rates are jointly nonstationary for all ten series over the sample period 1973-1987. Their results indicate a marginal rejection of the null hypothesis of joint nonstationarity at conventional nominal levels of significance and are interpreted as evidence in favor of PPP. The study of Abuaf and Jorion (1990) has stimulated a strand of literature which employs multivariate generalizations of unit root tests in order to increase the test power (e.g. Flood and Taylor, 1996; Wu, 1996; Frankel and Rose, 1996; Coakley and Fuertes, 1997; Lothian, 1997b; O’Connell, 1998; Papell, 1998). A number of these studies provide evidence supporting long-run PPP, given a sufficiently broad range of countries is considered and even only on post-Bretton Woods data.¹⁰

Taylor and Sarno (1998) and Sarno and Taylor (1998) argue, however, that the conclusions suggested by some of these studies may be misleading due to an incorrect interpretation of the null hypothesis of the multivariate unit root tests employed by Abuaf and Jorion and the subsequent literature. The null hypothesis in those studies is joint nonstationarity of the real exchange rates considered and hence rejection of the null hypothesis may occur even if only one of the series considered is stationary. Therefore, if rejection occurs when a group of real exchange rates is examined, then it may not be very informative and certainly it cannot be concluded that this rejection implies evidence supporting PPP for all them. On the basis of a large number of Monte Carlo experiments calibrated on dollar real exchange rates among the G5 countries, for example, Taylor and Sarno (1998) find that, for a sample size corresponding to the span of the recent float, the presence of a single stationary process together with three unit root processes led to rejection at the 5 percent level of the joint null hypothesis of non-stationarity in about 65 percent of simulations when the root of the stationary process was as large as 0.95, and on more than 95 percent of occasions when the root of the single stationary process was 0.9 or less.²¹

Taylor and Sarno (1998) employ two multivariate tests for unit roots which are shown - using Monte Carlo methods - to be relatively more powerful than traditional univariate tests using data for the G5 over the post-Bretton Woods period. The first test is based on a generalization of

---

¹⁰ Flood and Taylor (1996) find strong support for mean reversion towards long-run PPP using data on 21 industrialized countries over the floating rate period and regressing five-, ten- and twenty-year average exchange rate movements on average inflation differentials against the US.

²¹ Note that the artificial data generating process is calibrated on quarterly data, so that roots of this magnitude are plausible - see Taylor and Sarno (1998) for further details.

²¹ O’Connell (1998) points out an additional problem with panel unit root tests, namely that they typically fail to control for cross-sectional dependence in the data, and shows that this may lead to considerable size distortion, raising the significance level of tests with a nominal size of 5 percent to as much as 50 percent.
the augmented Dickey-Fuller test where, unlike in Abuaf and Jorion (1990), the autocorrelation coefficients are not constrained to be equal across countries and a more general AR(4) regression for each real exchange rate is considered. Although the null hypothesis is rejected, the test does not allow the authors to identify for how many and for which currencies PPP holds. The second test is based on an extension of the Johansen cointegration procedure, employed by the authors as a multivariate unit root test. Given that among a system of $N$ I(1) series, there can be at most $N - 1$ cointegrating vectors, if one can reject the hypothesis that there are less than $N$ cointegrating vectors among $N$ series, this is equivalent to rejecting the hypothesis of nonstationarity of all of the series. Put another way, the only way there can be $N$ distinct cointegrating vectors among $N$ series is if each of the series is I(0) and so is itself a cointegrating relationship.22 Thus, the null hypothesis under the Johansen procedure as applied by Taylor and Sarno is that there are $(N - 1)$ or less cointegrating vectors among the $N$ series concerned in the panel, which implies that at least one of them is nonstationary; rejection of the null in this case implies that all of the series in the panel are mean reverting. By rejecting this null hypothesis at the 1 percent nominal level of significance, Taylor and Sarno provide evidence that real exchange rates for the G5 constructed using the CPI price level are mean reverting during the recent floating period.

3.7 The PPP puzzle

In the previous two sections we have discussed the way in which researchers have sought to address the power problem in testing for mean reversion in the real exchange rate - either through long span studies or through panel unit root studies. As we made clear in our discussion, however, whether or not the long-span or panel-data studies do in fact answer the question whether PPP holds in the long run remains contentious. As far as the long-span studies are concerned, as noted in particular by Frankel and Rose (1996), the long samples required to generate a reasonable level of statistical power with standard univariate unit root tests may be unavailable for many currencies (perhaps thereby generating a 'survivorship bias' in tests on the available data - Froot and Rogoff, 1995) and, in any case, may potentially be inappropriate because of differences in real exchange rate behavior both across different historical periods and across different nominal exchange rate regimes (e.g. Baxter and Stockman, 1989; Hegwood and Papell, 1999). As for panel-data studies, the potential problem with panel unit root tests, highlighted by the Monte Carlo evidence of Taylor and Sarno (1998), is that the null hypothesis in such tests is generally that all of the series are generated by unit-root processes, so that the probability of rejection of the null hypothesis may be quite high when as few as just one of the series under consideration is a realization of a stationary process.

Even if, however, we were to take the results of the long-span or panel-data studies as having solved the first PPP puzzle, a second PPP puzzle then arises as follows. Among the long-span and panel-data studies which do report significant mean-reversion of the real exchange rate, there appears to be a consensus that the size of the half-life of deviations from PPP is about three to five years (Rogoff, 1996). If we take as given that real shocks cannot account for the major part of the short-run volatility of real exchange rates (since it seems incredible that shocks to real factors such as tastes and technology could be so volatile) and that nominal shocks can only have strong effects over a time frame in which nominal wages and prices are sticky, then a second PPP puzzle is the apparently high degree of persistence in the real exchange rate (Rogoff, 1996). Rogoff (1996) sums this issue up as follows: ‘The purchasing power parity puzzle then is this: How can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at

22This assumes that the underlying process must be either I(0) or I(1).
which shocks appear to damp out?‘.

Since Rogoff first noted the PPP puzzle in 1996, researchers have sought to address this as an additional issue in research on real exchange rates. Allowing for underlying shifts in the equilibrium dollar-sterling real exchange rate (Harrod-Balassa-Samuelson, HBS, effects) over the past two hundred years through the use of nonlinear time trends, for example, Lothian and Taylor (2000) suggest that the half-life of deviations from PPP for this exchange rate may in fact be as low as two and a half years.

Recently, Taylor (2001b) has shown that empirical estimates of the half life of shocks to the real exchange rate may be biased upwards because of two empirical pitfalls. The first pitfall identified by Taylor relates to temporal aggregation in the data. Using a model in which the real exchange rate follows an AR(1) process at a higher frequency than that at which the data is sampled, Taylor shows analytically that the degree of upward bias in the estimated half life rises as the degree of temporal aggregation increases - i.e. as the length of time between observed data points increases. The second pitfall highlighted by Taylor concerns the possibility of nonlinear adjustment of real exchange rates. On the basis of Monte Carlo experiments with a nonlinear artificial data generating process, Taylor shows that there can also be substantial upward bias in the estimated half life of adjustment from assuming linear adjustment when in fact the true adjustment process is nonlinear. The time aggregation problem is a difficult issue for researchers to deal with since, as discussed above, long spans of data are required in order to have a reasonable level of power when tests of nonstationarity of the real exchange rate are applied, and long spans of high-frequency data do not exist. On the other hand, Taylor also shows that the problem becomes particularly acute when the degree of temporal aggregation exceeds the length of the actual half life, so that this source of bias may be mitigated somewhat if the researcher believes that the true half life is substantially greater than the frequency of observation. In any case, the literature to date has only begun to explore the issue of nonlinearities in real exchange rate adjustment.

3.8 Nonlinear real exchange rate dynamics

The models discussed above in the context of determining the stochastic process of the deviation from the LOP (Section 2) also imply nonlinearity in the real exchange rate. In fact, they suggest that the exchange rate will become increasingly mean reverting with the size of the deviation from the equilibrium level. In some models the jump to mean-reverting behavior is sudden, whilst in others it is smooth, and Dumas (1994) suggests that even in the former case, time aggregation will tend to smooth the transition between regimes. Moreover, if the real exchange rate is measured using price indices made up of goods prices each with a different size of international arbitrage costs, one would expect adjustment of the overall real exchange rate to be smooth rather than discontinuous.

Michael, Nobay and Peel (MNP) (1997) and Taylor, Peel and Sarno (TPS) (2001) propose an econometric modelling framework for the empirical analysis of PPP which allows for the fact that commodity trade is not frictionless and for aggregation across goods with different thresholds. To state the issues clearly, recall that equilibrium models of exchange rate determination in the presence of transaction costs have been proposed by Benninga and Protopapadakis (1988), Dumas (1992) and Sercu, Uppal and van Hulle (1995). As a result of the costs of trading goods, persistent deviations from PPP are implied as an equilibrium feature of these models (deviations are left uncorrected as long as they are small relative to the costs of trading). A significant insight into the nature of PPP deviations is provided by Dumas (1992), who analyses the dynamic process of the real exchange rate in spatially separated markets under proportional transaction costs.
Deviations from PPP are shown to follow a nonlinear process which is mean reverting. The speed of adjustment towards equilibrium varies directly with the extent of the deviation from PPP. Within the transaction band, when no trade takes place, the process is divergent so that the exchange rate spends most of the time away from parity. This implies that deviations from PPP last for a very long time (p. 154), although they certainly do not follow a random walk.\footnote{Dumas (1992) conjectures that the Roll (1979) “ex ante PPP” hypothesis holds as a limiting case of his model as the degree of risk aversion tends to zero, although see Section 4 below.} \footnote{A framework also generating nonlinearities in real exchange rate dynamics may be based upon a model in which there are heterogeneous agents exercising influence in the foreign exchange market, namely economic fundamentalists, technical analysts and noise traders (Killian and Taylor, 2001). See Allen and Taylor (1990), Taylor and Allen (1992), and Sarno and Taylor (2001a), for a discussion of the importance of the influence of technical analysis in the foreign exchange market. Taylor and Kubelec (2001) also provide some evidence that official foreign exchange intervention may impart nonlinearity into real exchange rate movements.}

In the procedures conventionally applied to test for long-run PPP, the null hypothesis is usually that the process generating the real exchange rate series has a unit root, while the alternative hypothesis is that all of the roots of the process lie within the unit circle. Thus, the maintained hypothesis in the conventional framework assumes a linear autoregressive process for the real exchange rate, which means that adjustment is both continuous and of constant speed, regardless of the size of the deviation from PPP. As noted above, however, the presence of transaction costs may imply a nonlinear process which has important implications for the conventional unit root tests of long-run PPP. Some empirical evidence of the effect of transaction costs on tests of PPP is provided by Davutyan and Pippenger (1990). More recently, Obstfeld and Taylor (1997) have investigated the nonlinear nature of the adjustment process in terms of a threshold autoregressive (TAR) model (Tong, 1990) that allows for a transaction costs band within which no adjustment takes place while outside of the band the process switches abruptly to become stationary autoregressive. While discrete switching of this kind may be appropriate when considering the effects of arbitrage on disaggregated goods prices (Obstfeld and Taylor, 1997), discrete adjustment of the aggregate real exchange rate would clearly be most appropriate only when firms and traded goods are identical. Moreover, many of the theoretical studies discussed above suggest that smooth rather than discrete adjustment may be more appropriate in the presence of proportional transaction costs and, as suggested by Teräsvirta (1994), Dumas (1994) and Bertola and Caballero (1990), time aggregation and nonsynchronous adjustment by heterogeneous agents is likely to result in smooth aggregate regime switching.

An alternative characterization of nonlinear adjustment, which allows for smooth rather than discrete adjustment is in terms of a smooth transition autoregressive (STAR) model (Granger and Teräsvirta, 1993). This is the model employed by Michael, Nobay and Peel (1997) and Taylor, Peel and Sarno (2001). In the STAR model, adjustment takes place in every period but the speed of adjustment varies with the extent of the deviation from parity. A STAR model may be written:

\[ y_t - \mu = \sum_{j=1}^{p} \beta_j \left( y_{t-j} - \mu \right) + \sum_{j=1}^{p} \beta_j^* \left( y_{t-j} - \mu \right) + \Phi(\theta; y_{t-\ell} - \mu) + \varepsilon_t \]

where \( \{ q_t \} \) is a stationary and ergodic process, \( \varepsilon_t \sim iid(0, \sigma^2) \) and \( (\theta \mu) \in \{ \mathbb{R} \times \mathbb{R} \} \), where \( \mathbb{R} \) denotes the real line \( (-\infty, \infty) \) and \( \mathbb{R}^+ \) the positive real line \( (0, \infty) \). The transition function \( \Phi(\theta; y_{t-\ell} - \mu) \) determines the degree of mean reversion and is itself governed by the parameter \( \theta \), which effectively determines the speed of mean reversion, and the parameter \( \mu \) which is the equilibrium level of \( \{ q_t \} \). A simple transition function, suggested by Granger and Teräsvirta (1993), is the exponential function:

\[ \Phi(\theta; y_{t-\ell} - \mu) = 1 - \exp \left[ -\theta^2 (y_{t-\ell} - \mu)^2 \right] \],

(18)

where \( \theta \) is the degree of mean reversion.
in which case (17) would be termed an exponential STAR or ESTAR model. The exponential transition function is bounded between zero and unity, \( \Phi : \mathbb{R} \to [0, 1] \), has the properties \( \Phi[0] = 0 \) and \( \lim_{x \to \pm \infty} \Phi[x] = 1 \), and is symmetrically inverse—bell shaped around zero. These properties of the ESTAR model are attractive in the present modelling context because they allow a smooth transition between regimes and symmetric adjustment of the real exchange rate for deviations above and below the equilibrium level. The transition parameter \( \theta \) determines the speed of transition between the two extreme regimes, with lower absolute values of \( \theta \) implying slower transition. The inner regime corresponds to \( qt - \mu = \mu \), when \( \Phi = 0 \) and (17) becomes a linear AR(p) model:

\[
[q_t - \mu] = \sum_{j=1}^{p} \beta_j [q_{t-j} - \mu] + \varepsilon_t. \tag{19}
\]

The outer regime corresponds, for a given \( \theta \), to \( \lim_{[q_t - \mu] \to \pm \infty} \Phi[\theta; q_t - \mu] \), where (17) becomes a different AR(p) model:

\[
[q_t - \mu] = \sum_{j=1}^{p} (\beta_j + \beta^*_j) [q_{t-j} - \mu] + \varepsilon_t \tag{20}
\]

with a correspondingly different speed of mean reversion so long as \( \beta^*_j \neq 0 \) for at least one value of \( j \).

It is also instructive to reparameterize the STAR model (17) as

\[
\Delta q_t = \alpha + \rho q_{t-1} + \sum_{j=1}^{p-1} \phi_{j} \Delta q_{t-j} + \begin{cases} 
\alpha^* + \rho^* q_{t-1} + \sum_{j=1}^{p-1} \phi^*_j \Delta q_{t-j} \Phi[\theta; q_t - \mu] + \varepsilon_t, & \text{if } q_t - \mu = \mu, \\
\alpha + \rho^* q_{t-1} + \sum_{j=1}^{p-1} \phi^*_j \Delta q_{t-j} \Phi[\theta; q_t - \mu] + \varepsilon_t, & \text{if } q_t - \mu \neq \mu.
\end{cases}
\]

(21)

where \( \Delta q_{t-j} = q_{t-j} - q_{t-j-1} \). In this form, the crucial parameters are \( \rho \) and \( \rho^* \). Our discussion of the effect of transaction costs above suggests that the larger the deviation from PPP the stronger will be the tendency to move back to equilibrium. This implies that while \( \rho \geq 0 \) is admissible, we must have \( \rho^* < 0 \) and \( \rho + \rho^* < 0 \). That is, for small deviations \( q_t \) may be characterized by unit root or even explosive behavior, but for large deviations the process is mean reverting. This analysis has implications for the conventional test for a unit root in the real exchange rate process, which is based on a linear AR(p) model, written below as an augmented Dickey-Fuller regression:

\[
\Delta q_t = \alpha' + \rho' q_{t-1} + \sum_{j=1}^{p-1} \phi_{j} \Delta q_{t-j} + \varepsilon_t. \tag{22}
\]

Assuming that the true process for \( q_t \) is given by the nonlinear model (21), estimates of the parameter \( \rho' \) in (22) will tend to lie between \( \rho \) and \( \rho + \rho^* \), depending upon the distribution of observed deviations from the equilibrium level \( \mu \). Hence, the null hypothesis \( H_0 : \rho' = 0 \) (a single unit root) may not be rejected against the stationary linear alternative hypothesis \( H_1 : \rho' < 0 \), even though the true nonlinear process is globally stable with \( \rho + \rho^* < 0 \). Thus, failure to reject the unit root hypothesis on the basis of a linear model does not necessarily invalidate long-run PPP.

MNP (1997) apply this model to monthly interwar data for the French franc-US dollar, French franc-UK sterling and UK sterling-US dollar as well as for the Lothian and Taylor (1996) long span data set. Their results clearly reject the linear framework in favor of an ESTAR process. The systematic pattern in the estimates of the nonlinear models provides strong evidence of mean-reverting behavior for PPP deviations, and helps explain the mixed results of previous studies. However, the periods examined by MNP are ones over which the relevance of long run PPP is uncontentious (Taylor and McMahon, 1988; Lothian and Taylor, 1996; Lothian and McCarthy, 2000).
Using data for the recent float, however, TPS (2001) record empirical results that provide strong confirmation that four major real bilateral dollar exchange rates are well characterized by nonlinearly mean reverting processes over the floating rate period since 1973. Their estimated models imply an equilibrium level of the real exchange rate in the neighborhood of which the behavior of the log-level of the real exchange rate is close to a random walk, becoming increasingly mean reverting with the absolute size of the deviation from equilibrium, consistent with the recent theoretical literature on the nature of real exchange rate dynamics in the presence of international arbitrage costs. TPS also estimated the impulse response functions corresponding to their estimated nonlinear real exchange rate models by Monte Carlo integration.25 By taking account of statistically significant nonlinearities, TPS find the speed of real exchange rate adjustment to be typically much faster than the very slow speeds of real exchange rate adjustment hitherto recorded in the literature. These results therefore seem to shed some light on Rogoff’s PPP puzzle (Rogoff, 1996). In particular, it is only for small shocks occurring when the real exchange rate is near its equilibrium that the nonlinear models consistently yield half lives in the range of three to five years, which Rogoff (1996) terms ‘glacial’. For dollar-mark and dollar-sterling in particular, even small shocks of one to five percent have a half life under three years. For larger shocks, the speed of mean reversion is even faster.26 27

In a number of Monte Carlo studies calibrated on the estimated nonlinear models, TPS also demonstrate the very low power of standard univariate unit root tests to reject a false null hypothesis of unit root behavior when the true model is nonlinearly mean reverting, thereby suggesting an explanation for the difficulty researchers have encountered in rejecting the linear unit root hypothesis at conventional significance levels for major real exchange rates over the recent floating rate period. Panel unit root tests, however, displayed much higher power in their rejection of the false null hypothesis against an alternative of nonlinear mean reversion, in keeping with the recent literature. The results of TPS therefore encompass previous empirical work in this area.28

3.9 Do we care if the real exchange rate has a unit root?

A reading of this section is likely to convey the impression that what researchers have mainly done in this area of international finance is testing the hypothesis of nonstationarity. Indeed, this is the case to a large extent, but we believe that this is an exercise which delivers important economic implications. There are several reasons why we should care if the real exchange rate has a unit root. First, we should care because the degree of persistence in the real exchange rate can be used to infer what the principal impulses driving exchange rate movements are. In particular, if the real exchange rate is highly persistent (for example close to a random walk), then the shocks must be real-side, principally technology shocks, whereas if there is little persistence, then the shocks must

25Note that, because of the nonlinearity, the half-lives of shocks to the real exchange rates vary both with the size of the shock and with the initial conditions.

26Half-lives estimated using ESTAR models fitted to mark-based European real exchange rate series (Taylor and Sumo, 1999) were generally slightly lower than those for dollar-based real exchange rates. This is not surprising, given the proximity of the European markets involved and the fact that they are operating within a customs union, and accords with previous evidence on the mean-reverting properties of European real exchange rates (e.g. Canova, Cumby and Dilba, 1999; Cheung and Lai, 1998).

27In a complementary study, Taylor and Peel (2000) fit ESTAR models to deviations of the nominal exchange rate from the level suggested by ‘monetary fundamentals’, and find that the model performs well for dollar-mark and dollar-sterling over the recent float.

28In their fitted ESTAR models, the real exchange rate will be closer to a unit root process the closer it is to its long-run equilibrium. Somewhat paradoxically, therefore, failure to reject a unit root may indicate that the real exchange rate has on average been relatively close to equilibrium, rather than implying that no such long-run equilibrium exists.
be principally to aggregate demand, such as, for example, innovations to monetary policy (Rogoff, 1996). Second, from a theoretical perspective, nonstationarity of the real exchange rate implies that PPP is not a valid long-run international parity condition (Taylor, 1995). In turn, given that much open economy macroeconomics is based on the assumption of PPP, nonstationarity of the real exchange rate would imply that a large strand of open economy macroeconomic theory may be flawed due to this assumption. Indeed, it is well known that the implications of open economy dynamic models depend sensitively on the presence or absence of a unit root in the real exchange rate (Sarno, 2001). Third, estimates of PPP exchange rates are often used for practical purposes such as determining the degree of misalignment of the nominal exchange rate and the appropriate policy response, the setting of exchange rate parities, and the international comparison of national income levels. These practical uses of the PPP concept, and in particular the calculation of PPP exchange rates, would obviously be affected if the real exchange rate contains a unit root.\textsuperscript{29}

Overall, in our view, although the presence of a statistically and economically important unit root component in the real exchange rate may still be contentious, the above arguments may clarify why we should care if the real exchange rate has a unit root, which, in turn, justifies the efforts of researchers in international finance who have searched for stationarity of the real exchange rate for decades.\textsuperscript{30}

4 Modelling long-run PPP deviations

Modifications and extensions of the simple PPP hypothesis exist which try to rationalize the existence of long-run deviations from PPP. The most popular of these is the Harrod-Balassa-Samuelson model (Harrod, 1933; Balassa, 1964; Samuelson, 1964).

The underlying argument of the Harrod-Balassa-Samuelson model is as follows. Suppose - for the sake of argument - that the LOP holds among traded goods. In the fast-growing economy, productivity growth will tend to be concentrated in the traded goods sector. This will lead to wage rises in the traded goods sector without the necessity for price rises. Hence traded goods prices can remain constant and the LOP continues to hold with an unchanged nominal exchange rate. But workers in the non-traded goods sector will also demand comparable pay rises, and this will lead to an overall rise in the CPI. Since the LOP holds among traded goods and, by assumption, the nominal exchange rate has remained constant, this means that the upward movement in the domestic CPI will not be matched by a movement in the nominal exchange rate so that, if PPP initially held, the domestic currency must now appear overvalued on the basis of comparison made using CPIs expressed in a common currency at the prevailing exchange rate. The crucial assumption is that productivity growth is much higher in the traded goods sector.\textsuperscript{31} Note also that the relative price of nontradables may rise even in the case of balanced growth of the two sectors of the economy, as long as the nontraded goods sector is more labor intensive relative to the traded goods sector.

We can analyze this issue more formally using a simple small open economy model due to Froot and Rogoff (1995).\textsuperscript{32} Consider the following production functions for the two sectors of the

\textsuperscript{29}The title of this subsection is highly borrowed from the work of Christiano and Eichenbaum (1990), who raised the question of why and if researchers should care about the existence of a unit root in output.

\textsuperscript{30}See Rogoff (1996) for further discussion of these issues.

\textsuperscript{31}This is an argument first advanced and empirically tested by Baerel and Bowen (1966).

\textsuperscript{32}Rogoff (1996) extends this model adding a more interesting dynamics and also provides a more comprehensive treatment of the Harrod-Balassa-Samuelson effect; see also Obstfeld and Rogoff (1996, Chapter 4).
\[ Y^I = A^I \left(L^I\right)^{\theta^I} \left(K^I \right)^{1-\theta^I} \quad I = T, N \]  

where \( Y^I, K^I, L^I \) and \( A^I \) denote domestic output, capital, labor and productivity in the sector of the economy considered; the superscripts \( T \) and \( N \) denote the traded and nontraded sector respectively; time subscripts are omitted for simplicity. The model also assumes perfect factor mobility and perfect competition in both traded and nontraded sectors. Thus, the equations for the world (and domestic) interest rate and wages may be derived in terms of the endogenous and exogenous variables of the two sectors as follows:

\[ R = \left(1 - \theta^T\right) A^T \left(K^T / L^T\right)^{-\theta^T} \]  

(24)

\[ R = P^N \left(1 - \theta^N\right) A^N \left(K^N / L^N\right)^{-\theta^N} \]  

(25)

\[ W = \theta^T A^T \left(K^T / L^T\right)^{1-\theta^T} \]  

(26)

\[ W = P^N \theta^N A^N \left(K^N / L^N\right)^{1-\theta^N} \]  

(27)

where \( R \) denotes the world cost of capital, \( W \) is the wage rate measured in tradables and \( P^N \) is the relative price of the nontradable goods. The model (41)-(44) provides a solution for the four endogenous variables, that is the capital-labor ratios for the two sectors of the economy, the wage rate and the price level. Taking logs and totally differentiating (24)-(27), the model can be rewritten as follows:

\[ \hat{\alpha}^T - \theta^T \hat{k}^T = 0 \]  

(28)

\[ \hat{\theta}^N + \hat{\alpha}^N - \theta^N \hat{k}^N = 0 \]  

(29)

\[ \hat{\omega} = \hat{\alpha}^T + \left(1 - \theta^T\right) \hat{k}^T \]  

(30)

\[ \hat{\omega} = \hat{\theta}^N + \hat{\alpha}^N + \left(1 - \theta^N\right) \hat{k}^N, \]  

(31)

where the variables in lowercase are in logarithms and the hats denotes total differentials of the variables in question. Finally, the solutions for the endogenous variables of the model, obtained using (28)-(31), are:

\[ \hat{k}^N = \hat{k}^T = \hat{\omega} = \hat{\alpha} / \theta^T \]  

(32)

\[ \hat{\theta}^N = \left(\theta^N / \theta^T\right) \hat{\alpha}^T - \hat{\alpha}^N. \]  

(33)
According to equation (32), the model predicts that the percentage change in the capital-labor ratios are the same in the traded and nontraded goods sectors and they are also equal to the wage rate differential. Equation (33) incorporates the Harrod-Balassa-Samuelson condition: the percentage change of nontradables is determined only by the production side of the economy, while demand factors do not affect the real exchange rate in the long run. If the degree of labor and capital intensity is the same in the traded and nontraded sectors, i.e. $\theta_T = \theta^N$, then the percentage change in prices is exactly equal to the productivity differential between the two sectors. Nevertheless, if the nontraded sector is more labor intensive than the traded sector, i.e. $\theta^N > \theta_T$, then even in a situation of balanced productivity growth in the two sectors, the relative price of nontradables will rise. With one component of the CPI constant and the other one increasing, the overall price level must increase.\textsuperscript{33}

Japan is often referred to as a good example of the Harrod-Balassa-Samuelson effect in operation, since it has been on average the fastest-growing economy for much of the post World War II period. In the case of Japan, the effect is very significant regardless of the price index used even if, in general, one would expect the Harrod-Balassa-Samuelson effect to be relatively stronger when the CPI is used rather than the WPI, since the latter includes a higher component of tradables (see Lothian, 1990, 1991; Rogoff, 1996).\textsuperscript{34}

Apart from a few exceptions, however, the empirical evidence provides mixed results on the Harrod-Balassa-Samuelson effect. For example, Rogoff (1992) provides an alternative explanation of the near random walk hypothesis of the real exchange rate different than the near random walk behavior in the underlying fundamentals. He builds a neoclassical open-economy model with traded and nontraded goods, where agents are able to smooth their consumption of tradables over time through the international capital markets in the face of productivity shocks in the traded goods sector. In this model agents cannot, however, smooth productivity shocks in the nontraded goods sector, but these shocks are assumed not to be very significant, as suggested by some theory and empirical evidence (Baumol and Bowen, 1966). The implications and the empirical estimates of the model are in sharp contrast to the predictions of the Harrod-Balassa-Samuelson model, even if the exchange rate considered is the Japanese yen-US dollar during the recent float.\textsuperscript{35}

Some other recent empirical evidence exists, however, in favor of the Harrod-Balassa-Samuelson hypothesis. Heston, Nuxoll and Summers (1994) examine the tradable-nontradable price differential across countries on the basis of the International Comparison Program (ICP) data set and find, using a variety of regressions, that the difference between tradable and nontradable price parities move with income, consistent with the view of Harrod, Balassa and Samuelson.

A number of authors have also suggested that demand factors - notably real government consumption - may generate deviations from PPP if there is a bias towards the service sector, since

\textsuperscript{33} An alternative underlying theory which also leads to the Harrod-Balassa-Samuelson hypothesis is due to Kravis and Lipsey (1983) and Bhagwati (1984), who build an imperfect capital mobility model and hence use the assumption that capital-labor ratios are higher in fast-growing countries relative to slow-growing countries. For a comprehensive overview of the theoretical contributions and the empirical evidence on the Harrod-Balassa-Samuelson effect, see Asea and Corden (1994).

\textsuperscript{34} Also, Marston (1987) and Edison and Klocknad (1987) provide strong empirical evidence for the existence of a Harrod-Balassa-Samuelson effect for the Japanese yen-US dollar and the UK sterling-Norwegian krone exchange rates respectively. Similar findings are provided for both Germany and Japan by Hsieh (1982) and Obstfeld (1993).

\textsuperscript{35} Asea and Mendoza (1994) build a neoclassical general-equilibrium model which has similar implications to the Harrod-Balassa-Samuelson model. First, productivity differentials determine international differences in relative prices of nontradable goods. Second, deviations from PPP reflect differences in nontradable prices. The results from estimating the model on a panel of OECD countries provide empirical evidence that productivity differentials can explain low-frequency differences in relative prices. Nevertheless, predicted relative prices of nontradable goods cannot explain long-run deviations from PPP.
this will tend to raise the relative price of non-tradables. De Gregorio, Giovannini and Wolf (1994) estimate, using panel data methods, regressions of the form:

\[
(p^N - p^T)_{i,t} = \phi_1 + \varphi_1 \left[ \left( \theta^N / \theta^T \right) a^T - a^N \right]_{i,t} + \varphi_2 g_{i,t} + \varphi_3 y_{i,t},
\]

where \( g \) denotes the ratio of real government spending (excluding government investment) to real GDP, \( y \) is real income per capita, and \( i \) is a country subscript. Their empirical results for a panel of fourteen OECD countries suggest that productivity, government spending and income are all important variables in explaining the tradable-nontradable price differential and the parameters \( \varphi_1, \varphi_2 \), and \( \varphi_3 \) are all statistically significant at conventional nominal levels of significance and correctly signed.\(^{36}\) In order to investigate the long-run significance of the demand factors (\( g \) and \( y \)), De Gregorio, Giovannini and Wolf also estimate the same regression on average data for the same panel of countries; their results suggest that demand factors become less important over the long run.\(^{37}\) De Gregorio and Wolf (1994) decompose the component of real exchange rate movements determined by the Harrod-Balassa-Samuelson effect and the component caused by changes in the relative prices of traded goods, that is changes in the terms of trade. They find that the latter appears to be more important than the Harrod-Balassa-Samuelson effect in explaining short-term real exchange rate movements. On the other hand, Chinn (2000), in an analysis of a set of Asia-Pacific economies, finds that neither government spending nor the terms of trade appear to be important factors. Also, Rogers and Jenkins (1995) and Engel (1999) present evidence that the Harrod-Balassa-Samuelson effect does not seem evident in CPI-based real exchange rates, even at long horizons.

Overall, the empirical evidence on the Harrod-Balassa-Samuelson effect is quite mixed. Even if some evidence exists that the productivity differential is an important factor in explaining the tradable-nontradable price differential and the real exchange rate, increasingly strong evidence supporting long-run convergence to PPP is provided by the recent literature, perhaps simply because technological progress is mobile across borders in the very long run.\(^{38}\)

5 The source of shocks to real and nominal exchange rates

A related strand of the empirical literature on real exchange rate behavior has investigated the source of disturbances (shocks) to real and nominal exchange rates. Much of this evidence comes from employing vector autoregression analysis inspired by the methodology originally developed by Blanchard and Quah (hereafter BQ, 1989).\(^{39}\) In this section we provide a brief review of this literature.

\(^{36}\)Data for productivity are usually computed in this literature using Solow residuals.

\(^{37}\)Alesina and Perotti (1995) argue that fiscal policy may also have long run real effects if distortionary taxes are used in order to finance government spending programs.

\(^{38}\)Some researchers also argue that a strong long-run relationship exists between persistent deficits in the current account balance and the depreciation of the real exchange rate. In fact, a close correlation is often found between these two variables. For example, Obstfeld and Rogoff (1995) find a large correlation coefficient between trade-weighted real exchange rate changes and changes in net foreign asset positions for fifteen countries during the 1980s. Correlation between these two endogenous variables does not necessarily imply, however, that there is causation. For example, Bayoumi, Clark, Sjahansky and Taylor (1994) use the International Monetary Fund’s MULTIMOD and provide evidence that the correlation between current account deficits and the real exchange rate is in fact very sensitive to whether the driving factor is fiscal or monetary policy. More generally, current account deficits may be rationalized on the basis of many different driving factors.

\(^{39}\)BQ (1989) provide an econometric technique which allows researchers to decompose a time series into its temporary and permanent components in the context of a vector autoregression. In their study, BQ decompose real output into its transitory and permanent components. They motivate their empirical analysis using a stylized
Lastrapes (1992) applies the BQ decomposition to real and nominal US dollar exchange rates of five industrialized countries (Germany, UK, Japan, Italy and Canada) over the sample period from 1973 to 1989. Lastrapes finds that real shocks cause a permanent real and nominal appreciation, while nominal shocks are found to cause a permanent nominal depreciation. Evans and Lothian (1993) also examine real dollar exchange rates of four major industrial countries under the recent float and report that transitory shocks play a relatively small role in explaining real exchange rate movements. Their findings, however, also suggest that there are instances when temporary shocks make a more substantial contribution, so that the role of temporary and permanent shocks in driving exchange rate movements may be varying over time. Enders and Lee (1997) discover similar findings in an investigation of the sources of real and nominal exchange rate fluctuations for several major industrialized countries over the post-Bretton Woods period. In a highly cited study, Clarida and Gali (1994) estimate the relative contribution of three types of shocks to four major real US dollar exchange rates during the post-Bretton Woods era. Clarida and Gali assume that one type of shock affects both the real exchange rate and output in the long run. They interpret this shock as a supply shock. Clarida and Gali further assume that another shock only affects the real exchange rate in the long-run but not output, and they label this as a demand shock. Finally, all shocks that influence neither the long-run real exchange rate nor output are denoted as monetary shocks. Clarida and Gali find that monetary shocks account for 41 percent of the unconditional variance of changes in the real German mark rate and 35 percent of the unconditional variance of changes in the real Japanese yen rate. They also estimate that monetary shocks account for no more than 3 percent of the unconditional variance of real exchange rate changes over all horizons for the UK sterling and Canadian dollar. Rogers (1999), however, employing a structural vector autoregression on a long-term data set provides evidence that monetary shocks account for almost one-half of the forecast error variance of the real dollar-sterling exchange rate over short horizons. In an earlier study, Eichenbaum and Evans (1995) also investigate the effects of shocks to US monetary policy on exchange rates. Specifically, they consider three measures of these shocks: orthogonalized shocks to the federal funds rate, orthogonalized shocks to the ratio of nonborrowed to total reserves, and changes in the Romer and Romer index of monetary policy. In contrast to a large literature, Eichenbaum and Evans find substantial evidence of a link between monetary policy and exchange rates.40

One problem with this literature is that identification restrictions are required in the vector autoregression model in order to identify uniquely the model and to generate impulse response functions and variance decompositions. Cushman and Zha (1997) argue that many empirical studies on the effects of monetary policy shocks in small open economies have generated puzzling dynamic responses in various macroeconomic variables due to an identification of monetary policy that is inappropriate for such economies. To remedy this, Cushman and Zha propose that a structural model be estimated to account explicitly for the features of the small open economy. Such a model is applied to Canada and is shown to generate dynamic responses to the identified monetary policy shock that are consistent with standard theory and highlight the exchange rate as a transmission mechanism.41 More recently, Kim and Roubini (2000) have again emphasized how past empirical research on the effects of monetary policy in closed and open economies has

---

40In particular, their results suggest that a contractionary shock to US monetary policy leads to (a) persistent, significant appreciations in US nominal and real exchange rates and (b) significant, persistent deviations from uncovered interest rate parity in favor of US interest rates.

41See also Prasad (1999), who develops an empirical framework for analysing the dynamics of the trade balance in response to different types of macroeconomic shocks.
found evidence of several anomalies, including the “exchange rate” puzzles. Kim and Roubini
develop an approach that provides a solution to some of these empirical anomalies in an open
economy setup. They use a structural vector autoregression with non-recursive contemporaneous
restrictions and identify monetary policy shocks by modeling the reaction function of the monetary
authorities and the structure of the economy. Their empirical findings are that effects of non-US
G7 monetary policy shocks on exchange rates and other macroeconomic variables are consistent
with the predictions of a broad set of theoretical models. The evidence is consistent with signif-
ificant, but transitory, real effects of monetary shocks. They also find that initially the exchange
rate appreciates in response to a monetary contraction; but after a few months, the exchange
rate depreciates over time in accordance with the uncovered interest parity condition. In a re-
lated study, Faust and Rogers (1999) start from stating that much empirical work addressing the
role of monetary policy shocks in exchange rate behavior has led to conclusions that have been
cloaked by the lack of plausible identifying assumptions. Faust and Rogers apply a statistical
procedure that allows them to relax identifying assumptions which they view as dubious. Their
work overturns some earlier results and strengthens others: i) contrary to some earlier findings
of “delayed overshooting” of the exchange rate, the peak exchange rate effect of policy shocks
may come nearly immediately after the shock; ii) monetary policy shocks lead to large uncovered
interest rate parity deviations; iii) monetary policy shocks may account for a smaller portion of the
variance of exchange rates than found in earlier studies. Faust and Rogers conclude that, while (i)
is consistent with overshooting, (ii) implies that the overshooting phenomenon cannot be driven
by Dornbusch’s (1976) mechanism, and (iii) gives reason to doubt whether monetary policy shocks
are the main source of exchange rate volatility. Another interesting extension of this literature is
the study by Kim (2001), who develops a structural vector autoregression to analyze jointly the
effects of foreign exchange policy (setting exchange reserves) and conventional monetary policy
(setting money and the interest rate) on the exchange rate, two types of policy reactions to the
exchange rate, and interactions between the two types of policies. Kim finds several interactions
among the two types of policies and the exchange rate, confirming the need for a joint analysis.
He also finds that foreign exchange policy has significant stabilizing effects on the exchange rate,
suggesting that it may be important to model foreign exchange policy explicitly when modeling
exchange rate behavior.

Although the majority of studies in this literature focus on major industrialized countries, there
are a few studies investigating developing and newly developed economies. The evidence provided
by this literature is clear: movements in real exchange rates of these economies are largely driven
by real shocks. For example, Chen and Wu (1997) focus on real exchange rate movements of four
Pacific Rim countries, providing evidence of the key role played by permanent shocks in explaining
the variability of the real exchange rates examined. Chen and Wu also find that real innovations
account for more than 90 percent of variations in the real Korean won at all time horizons. More
recently, Hoffmister and Roldos (2001) analyze the real Korean won and Brazilian real vis-à-vis
the US dollar and illustrate that temporary shocks can hardly explain any real exchange rate
movements.

Overall, the results provided by the literature on identifying the source of shocks driving real
and nominal exchange rates has provided mixed results. While this literature suggests that both
nominal (e.g. monetary) and real shocks explain both nominal and real exchange rate movements,
the relative importance of nominal and real shocks varies across studies when the exchange rates
examined involve major industrialized countries. With regard to developing and newly de-
veloped economies, however, although there have been relatively fewer studies, there seems to be a
consensus that real exchange rates are largely driven by real shocks.

26
Note that, despite the mixed findings of this literature, at least for real exchange rates among developed countries, the implicit assumption underlying Rogoff’s (1996) “PPP puzzle” (see Section 3.7) is that real exchange rates must be largely driven by nominal shocks, at least in the short run, in order to account for their high volatility. This is an area which would clearly repay further research.42

6 Conclusion

Within the vast literature on PPP and real exchange rates, professional opinion concerning the validity of PPP between the currencies of the major industrialized countries, in both the short and long run, appears to have shifted several times in the post war period. If there is an emerging consensus at the present time, it is probably reverting towards the view that long-run PPP does have some validity, at least for the major exchange rates, although a number of puzzles have yet to be resolved conclusively.

In our view, a promising strand of research which goes some way towards resolving both fundamental puzzles in this literature - namely whether PPP holds and whether one can reconcile the persistence of real exchange rates with their observed high volatility - has investigated the role of nonlinearities in real exchange rate adjustment toward long-run equilibrium. For example, Taylor, Peel and Sarno (2001) provide evidence of nonlinear mean reversion in a number of major real exchange rates during the post Bretton Woods period such that real exchange rates behave more like unit root processes the closer they are to long-run equilibrium and, conversely, become more mean reverting the further they are from equilibrium. Moreover, while small shocks to the real exchange rate around equilibrium will be highly persistent, larger shocks mean revert much faster than the ‘glacial rates’ previously reported for linear models (Rogoff, 1996). Further, Taylor, Peel and Sarno reconcile these results with the huge literature on unit roots in real exchange rates through Monte Carlo studies and, in particular, demonstrate that when the true data generating process implies nonlinear mean reversion of the real exchange rate, standard univariate unit root tests will have very low power, while multivariate unit root tests will have much higher power to reject a false null hypothesis of unit root behavior.

Further work on real exchange rate behavior might usefully be addressed to unraveling the relative contribution of prices and nominal exchange rates to movements in real exchange rates (see, for example, the recent study by Engel and Morley, 2001). This might be done, for example, in the context of nonlinear vector error correction models of the nominal exchange rate and domestic and foreign prices and other variables.43 Such a framework might also be extended to allow for the relative impact of monetary and fiscal policy on real exchange rate movements to be isolated and whether stronger evidence may be adduce for the Harrod-Balassa-Samuelson effect. Finally, the implications of nonlinearities in real exchange rate movements for exchange rate forecasting and, in turn, the influence of official exchange rate intervention in generating exchange rate nonlinearities, have yet to be fully examined.44

42Note also that many of these studies implicitly assume a unit root in the real exchange rate. Extending this research under the maintained hypothesis of a stationary real exchange rate would be a worthwhile avenue for future research.
44See Kilian and Taylor (2001); Clarida, Sarno, Taylor and Valente (2001); Sarno, Taylor and Peel (2001); and Taylor (2001).
References


Christiano, L.J. and M. Eichenbaum (1990), “Unit Roots in Real GNP: Do We Know, and Do We Care?”, Carnegie-Rochester Conference Series on Public Policy, 32, pp. 7-61.


